

MAGNETIC FIELD CALCULATIONS FOR HIGH-ENERGY PULSED POWER SUPPLIES

by

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ABSTRACT

The accurate calculation of the magnetic fields steady or rapidly varying is extremely important in designing pulsed power supplies for controlled thermonuclear fusion experiments, lasers, etc., where the traditional simplifying assumptions become unacceptable - especially when ferromagnetic materials in high magnetic fields are used.

A finite element method for solution of Maxwell's equations for a moving media in terms of the magnetic vector potential and electrokinetic scalar potential describing the penetration of the magnetic fields in fast pulsing power supplies of electromechanical type is presented. The formulation for the steady-state magnetic fields in nonlinear media results as a particular case of the method.

This approach was used for predicting the discharge parameters for the very fast discharging homopolar machine (FDX) designed by the Energy Storage Group at the University of Texas. FDX is in an advanced state of execution.

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Introduction

The generation of powerful current pulses by pulsed power supplies is intimately related to high magnetic fields accompanying them and to the penetration of such currents and diffusion of such magnetic fields into metallic current paths of the generator, transmission line and load.

An accurate prediction of the space and time distribution of magnetic fields is extremely important for the design of pulsed devices.

Finite element methods have been widely used since 1970, in solving steady state problems of magnetic fields, in complex, nonlinear systems [1,2,3].

In such situations the nonlinearity of the ferromagnetic materials is handled by linearizing the problem locally, solving the resulting linear equations and introducing the nonlinearity of the magnetization curve by an iterative correction to the linearized equations. Of course this assumes that the curve $B = f(H)$ is single valued, neglecting hysteresis effects.

The solution of the diffusion of magnetic fields by a finite element method - Galerkin projective technique was attempted in several papers [4,5]. Foggia et al [4] assumed a steady state regime and a prescribed sinusoidal travelling field impressed to the physical model. Miya [5] has a similar treatment prescribing the variation of plasma current in time.

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Actually it is not the solution for the forced response of the system but the solution for the "natural response" or rather the "complete response" that is required for pulsed power systems. The finite element method presented herein solves the transient diffusion problem in terms of a magnetic vector potential A through an intermediary electrokinetic scalar potential Ψ which transforms the forcing function concentrated in the generator into a distributed forcing function which acts locally in each point of the system: generator, transmission line, and load.

The physical problem

The physical problem treated herein is the discharging of a very fast homopolar generator which is a variant of the FDX experimental machine presented in detail in a companion paper at this conference. The scope of FDX is to explore the fundamental limitations of fast discharging homopolar power supplies[6].

The machine consists of two counterrotating aluminum rotors, storing at rated speed a kinetic energy of 0.36 MJ.

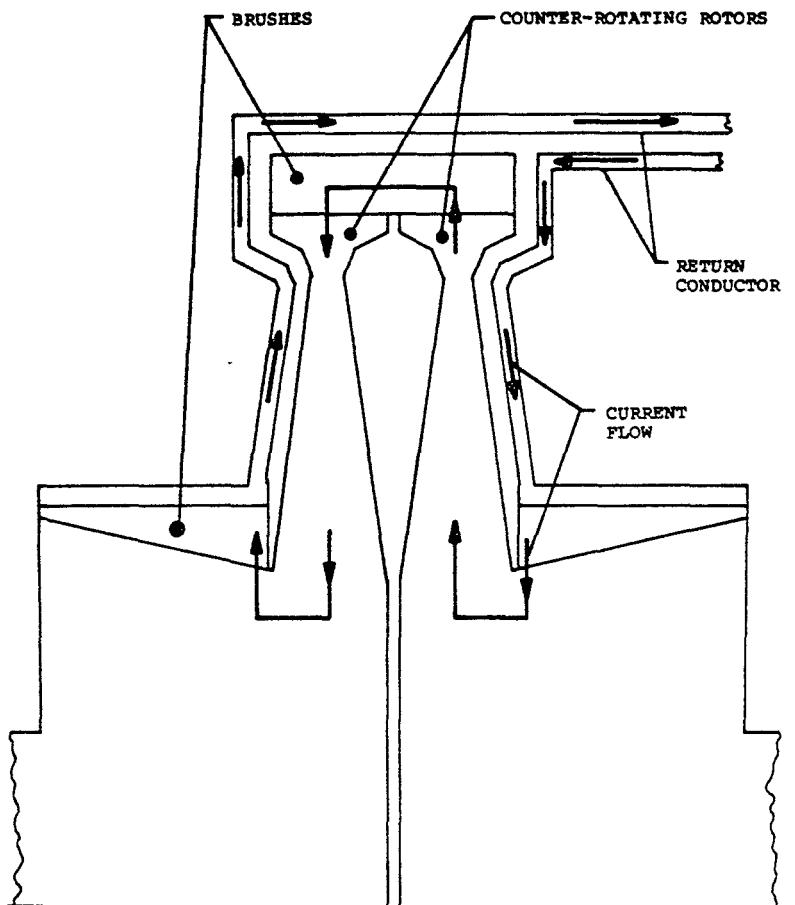


Figure 1: Machine configuration

The excitation is provided by a four turns field coil pulsed by discharging a 5 MJ slow discharge homopolar generator. The duration of the excitation pulse is long compared with the time of discharge of the FDX machine (the pulse has almost a flat top for 120 milliseconds). For this reason the flux density of the excitation field is assumed to be constant.

in time. The flux linking the active area of a rotor is 0.218 Webers at an average flux density of 4 Tesla which corresponds to a no-load voltage of 104 V per rotor, 208 V per machine calculated as line integral of $\int \mathbf{V} \times \mathbf{B}_0$.

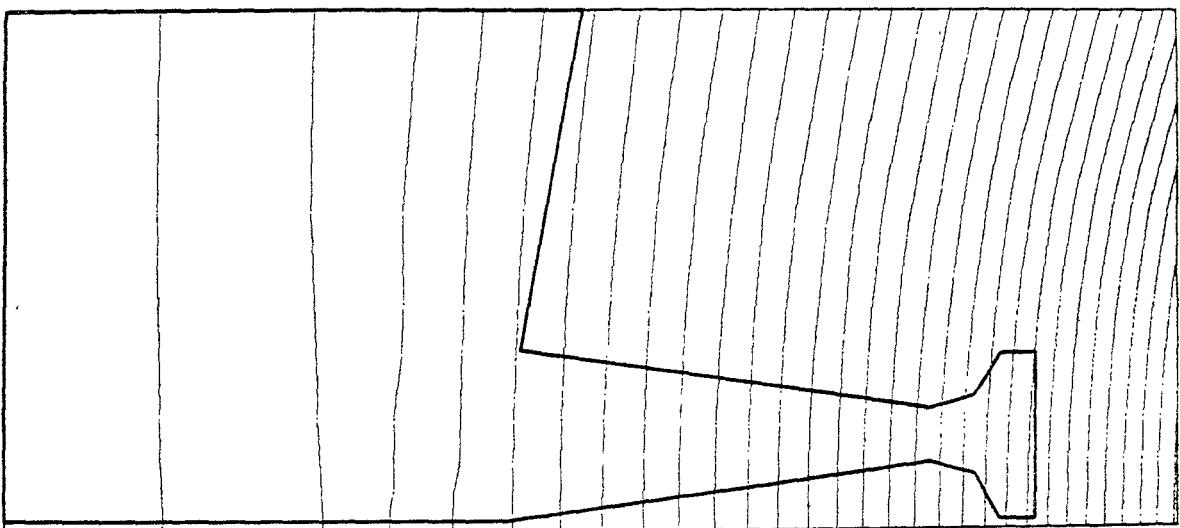


Figure 2: Excitation flux lines in rotor

Figure 2 shows a map of excitation flux linking the rotor, at the moment when the peak value is reached. At this time the brushes are applied to the rotor and a moment later the making switch is closed. The discharge current begins to flow, at the beginning, in a very thin layer or rotor and compensation plates, moments later penetrating into the interior of conductors.

The $\mathbf{J} \times \mathbf{B}_0$ forces stop the rotor, accelerate it in the opposite direction, stop it again until the stored energy is dissipated in space and time of the electromagnetic field in the process described above is the subject of this work.

The mathematical electromagnetic problem

The time scale of the electromagnetic phenomena concerning this particular problem the assumption of a quasistationary state holds and Maxwell's equations are used accordingly.

The energy stored at $t = 0$ in different parts of the system expressed through the electromagnetic state variable represents the input for Maxwell's equations, and the "natural response" of this system is described by the solution sought through the finite element approach.

The aforementioned treatments of the diffusion of electromagnetic field [4,5] were seeking the "forced response" of the system. Boundary conditions are given at all times in such cases, and through them, for given material properties of the system - the electromagnetic field quantities are uniquely defined.

Maxwell's equations, for a quasistationary regime, can be written as

$$(a) \quad \nabla \times \mathbf{H} = \mathbf{J}$$

$$(b) \quad \nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t} \quad (1)$$

$$(c) \quad \nabla \cdot \mathbf{B} = 0$$

Ohm's law in a moving media is given by

$$(d) \quad \mathbf{J} = \sigma (\mathbf{E} + \mathbf{V} \times \mathbf{B})$$

and, in view of the assumption of homogeneous, isotropic and hysteresis free materials

$$(e) \quad \mathbf{B} = \mu \mathbf{H}$$

In order to distribute the effects of the stored energy in the rotor characterized by the line integral of $\nabla \times \mathbf{B}$ as a variable of state, we introduce the electrokinetic scalar potential Ψ as an intermediate variable. In the moment of closing the switch, the electrokinetic state, characterized by Ψ is transmitted (with the speed of light) to each point of the closed electrical circuit.

This choice forces us to select as primary independent variable the magnetic vector potential \mathbf{A} , leading to

$$(f) \quad \mathbf{E} = - \nabla \Psi - \frac{\partial \mathbf{A}}{\partial t}$$

$$(g) \quad \mathbf{B} = \nabla \times \mathbf{A}$$

To these equations we adjoin the conditions that $\operatorname{div} \mathbf{J} = 0$, which expresses the conservation of charge condition and $\operatorname{div} \mathbf{A} = 0$.

Substitution and combining the above equations yields the set

$$(h) \quad \begin{aligned} \sigma \dot{\mathbf{A}} + \nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} - \sigma (\mathbf{V} \times \nabla \times \mathbf{A} - \nabla \Psi) &= \mathbf{0} \\ \nabla \cdot \sigma \nabla \Psi - \nabla \cdot \sigma (\mathbf{V} \times \nabla \times \mathbf{A} - \dot{\mathbf{A}}) &= 0 \end{aligned}$$

subject to

$$(i) \quad \nabla \cdot \mathbf{A} = 0.$$

Because both the rotor motion \mathbf{V} and the potential of the applied magnetic field \mathbf{A}_0 are axisymmetric and have only θ -components, the problem separates into two weakly coupled equations, (7) which governs the distribution of the steadily applied field and (6) in which the vector \mathbf{A} has only r and z components.

$$(j) \quad \nabla \times \frac{1}{\mu} \nabla \times \mathbf{A}_0 = \mathbf{J}_0$$

The finite element solution of (7), although necessary to our problem is a standard one and need not be discussed further here.

In addition to the equations (6), we have as side conditions, the vanishing of the potentials at large distances from the conductors and the usual continuity or jump conditions on current and field. Because the current distributions do not extend to infinity:

$$A_r = A_z = \Psi = 0 \quad \text{as } r \rightarrow \infty \quad (8)$$

and

$$\begin{aligned} \mathbf{n} \times \left[\frac{1}{\mu} (\nabla \times \mathbf{A}) \right] &= 0 \\ \mathbf{n} \cdot \left[\sigma (\nabla \times \nabla \times \mathbf{A}_0 - \nabla \Psi - \dot{\mathbf{A}}) \right] &= 0 \end{aligned} \quad (9)$$

In (9) the brackets denote the jump in the enclosed quantities across an interface between different materials or between moving and stationary materials. In the finite element model of the region shown in Figure 1, the symmetry about $r = 0$ and the antisymmetry about $z = 0$ give rise to the following additional boundary conditions

$$A_r = 0 \quad \text{on } r = 0 \quad (10)$$

$$A_r = \Psi = 0 \quad \text{on } z = 0.$$

Finite Element Formulation

We seek solutions to (6) and (9) subject to the conditions (8) and (10) in the weak or Galerkin sense. To this end we introduce a Lagrange multiplier λ corresponding to the constraint (6c) and the test functions u , η , and τ corresponding to \mathbf{A} , Ψ , and λ . The weak statement of the problem is

$$\sum_i \int_{\Omega_i} \left\{ \int_{\Gamma_i} [\sigma u \cdot \dot{\mathbf{A}} + \frac{1}{\mu} (\nabla \times \mathbf{A}) \cdot (\nabla \times u) + \sigma u \cdot \nabla \Psi + \lambda \nabla \cdot u + \tau \nabla \cdot \mathbf{A} - \sigma u \cdot (\nabla \times \nabla \times \mathbf{A}_0)] da \right\} = 0 \quad (11)$$

and

$$\sum_i \int_{\Omega_i} \left\{ \int_{\Gamma_i} [\sigma \nabla \eta \cdot \nabla \Psi + \sigma \dot{\mathbf{A}} \cdot \nabla \eta - \sigma (\nabla \times \nabla \times \mathbf{A}_0) \cdot \nabla \eta] da \right\} = 0$$

for arbitrary u , η , and τ vanishing at ∞ .

It is significant that the jump terms have equilibrated the boundary integrals from the integration by parts, i.e. the jump conditions are natural conditions that follow from (11).

The discrete form of (11) is obtained by introducing the finite element approximation of the fields \mathbf{A} , u , Ψ , η , λ and τ . The elements are quadrilaterals and the shape functions are quadratic for \mathbf{A} , u , Ψ , and η and linear for λ and τ . Thus each corner node of the mesh has a total of four unknowns, i.e. A_r , A_z , Ψ and λ ; while the midside nodes have only A_r , A_z , and Ψ . If the set of nodal point values is denoted \mathbf{q} then the discrete form of (11) can be written as

$$\sum_{\mathbf{q}} \mathbf{C} \dot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{f} \quad (12)$$

In (12) \mathbf{C} and \mathbf{K} are, for an element, 28×28 matrices whose elements are

the integrals of products of shape functions. It is apparent from (11) that neither C nor K is symmetric. The forcing function f in (12) contains terms proportional to $\nabla \times \nabla \times A$. The motion of the rotor is treated as given in this analysis and the value of the applied field A is known from an initial finite element solution for the single component $A_{\theta\theta}$.

Results

The first step in the analysis is to calculate steady excitation field B in which the discharge takes place. The forcing function responsible for discharge (energy conversion and transfer), $[\nabla \times B]$ has radial and axial components corresponding to the so-called "disk" and "drum" configurations of homopolar machines.

The z component is very small when compared to the radial one but in the area covered by brushes it can cause important circulating currents along the rotor-brush interface (Figures 3, 4).

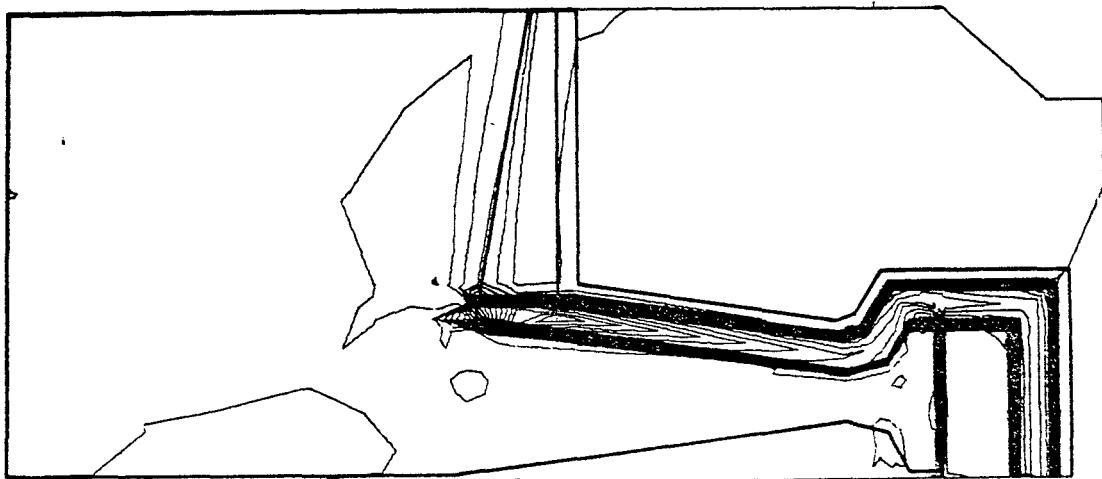


Figure 3: Magnetic field B_θ $t = 5 \mu\text{sec.}$

Figures 3 and 4 show the contours of B_θ at an early and late time during discharge. The current density vector J with no component in the θ direction produces a flux density with only one component. J is tangent to the contours and its magnitude is inversely proportional to the distance between them.

At $5 \mu\text{sec.}$ all the current, hence all the field, is confined to the first layer of elements adjacent to the airgap. Later (Figure 4) the current and field penetrate gradually - tending toward a uniform distribution. At the final time the rotor has stopped and begins to reverse. The $J \times B (\text{N/m}^3)$, electromagnetic force densities, and $\rho J^2 (\text{W/m}^3)$, specific ohmic loss, are easily available from the field distributions. From them design parameters as stresses and heating as functions of time can be calculated.

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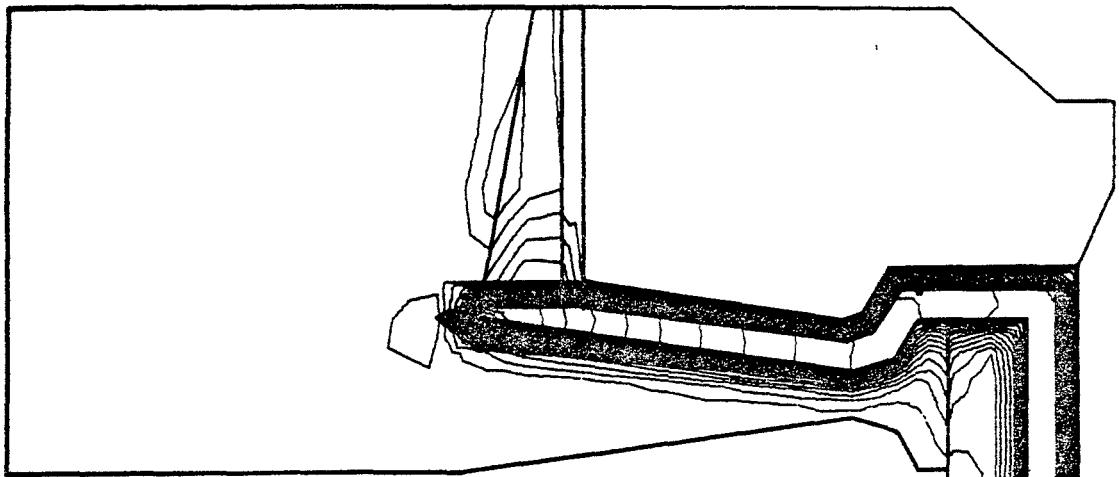


Figure 4: Magnetic field B_θ $t = 250 \mu\text{sec.}$

References

- ¹ Silvester, P. and Chari, M.V.K., "Finite-element solution of saturable magnetic field problems," IEEE Trans. on Power App. and Syst., vol. PAS-89, 1970, pp. 1642-1651.
- ² Chari, M.V.K. and Silvester, P., "Finite element analysis of magnetically saturated d.c. machines," IEEE Trans. on Power App. and Syst., vol. PAS-90, 1971, pp. 2362-2372.
- ³ Glowinski, R., and Marrocco, A., "Analyse numérique du champ magnétique d'un alternateur par éléments finis et sur-relaxation ponctuelle non linéaire," Computer methods in applied mechanics and engineering 3 (1974) pp. 55-85.
- ⁴ Foggia, A., Sabonnadiere, J. C., and Silvester, P., "Finite element solution of saturated travelling magnetic field problems," IEEE Trans. on Power App. and Syst., vol. PAS-94, No. 3, 1975, pp. 866-870.
- ⁵ Miya, K., An, S., Ando, Y., and Suzuki, Y., "Application of finite element method to electro-magneto-mechanical dynamics of superconducting magnetic coil and vacuum vessel," Proceedings of the 6th Symposium on Engineering Problems of Fusion Research, San Diego, Cal., Nov., 1975.
- ⁶ Driga, M. D., Nasar, S. A., Rylander, H. G., Weldon, W. F., and Woodson, H. H., "Fundamental limitations and topological considerations for fast discharge homopolar machines," IEEE Trans. on Plasma Science, vol. PS-3, No. 4, December, 1975, pp. 209-215.